Identities Related to Square Roots

If p and q are two positive real numbers

1. $\sqrt{pq} = \sqrt{p}\sqrt{q}$
2. $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$
3. $(\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q}) = p - q$
4. $(p + \sqrt{q})(p - \sqrt{q}) = p^2 - q$
5. $(\sqrt{p} + \sqrt{q})(\sqrt{r} + \sqrt{s}) = \sqrt{pr} + \sqrt{ps} + \sqrt{qr} + \sqrt{qs}$
6. $(\sqrt{p} + \sqrt{q})^2 = p + 2\sqrt{pq} + q$

Examples:

1. Simplify $(3 + \sqrt{7})(5 - \sqrt{11})$

We will use the identity

$$(\sqrt{p} + \sqrt{q})(\sqrt{r} + \sqrt{s}) = \sqrt{pr} + \sqrt{ps} + \sqrt{qr} + \sqrt{qs}$$
$$(3 + \sqrt{7})(5 - \sqrt{11}) = 15 + 5\sqrt{7} + 3\sqrt{11} + \sqrt{77}$$

2. Simplify $(\sqrt{5} + \sqrt{11})(\sqrt{5} - \sqrt{11})$

We will use the identity

$$(\sqrt{p} + \sqrt{q})(\sqrt{p} - \sqrt{q}) = p - q$$
$$(\sqrt{5} + \sqrt{11})(\sqrt{5} - \sqrt{11}) = 5 - 11 = -6$$

Laws of Exponents for Real Numbers

If we have a and b as the base and m and n as the exponents, then

1. $a^m \times a^n = a^{m+n}$
2. $(a^m)^n = a^{mn}$

3. $\frac{a^m}{a^n} = a^{m-n}, m > n$

4. $a^m b^m = (ab)^m$

5. $a^0 = 1$

6. $a^1 = a$

7. $1/a^n = a^{-n}$

1. **Let a > 0 be a real number and n a positive integer.**

   Then $\sqrt[n]{a} = b$, if $b^n = a$ and $b > 0$

   $$\sqrt[n]{a} = a^{\frac{1}{n}}$$

2. **Let a > 0 be a real number. Let m and n be integers such that m and n have no common factors other than 1, and n > 0. Then,**

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$